

This is an introductory text, in three chapters, dealing with the following topics: (i) Calculus of finite differences and difference equations, (ii) Numerical solution of ordinary differential equations, (iii) Numerical solution of partial differential equations. The term "simulations" in the title is to be understood in the restricted sense of simulating differential equations by finite-difference equations. An attempt has been made to incorporate recent advances in this field, particularly concerning the theory of error propagation and stability. Each chapter is followed by a short list of references and an extensive set of problems.

The text provides, at a modest level, a well-motivated introduction to the approximate solution of differential equations, and should serve well to prepare the student for a study of more specialized treatises on the subject.

W. G.

45[7, 9].—W. A. BEYER, N. METROPOLIS & J. R. NEERGAARD, *Square Roots of Integers 2 to 15 in Various Bases 2 to 10: 88062 Binary Digits or Equivalent*, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, December 1968. Plastic-bound computer printout, 277 pages, deposited in the UMT file.

The first ten tables here list \sqrt{n} for $n = 2, 3, 5, 6, 7, 10, 11, 13, 14$, and 15 to 29354 octal digits.

The next five tables list $\sqrt{2}$ to the bases 3, 5, 6, 7, and 10 to the equivalent accuracies: 55296, 36864, 32768, 30720, and 24576 digits, respectively. The last ten tables give the corresponding data for $\sqrt{3}$ and $\sqrt{5}$. Thus, starting on page 237, we find

$$(\sqrt{5})_5 = 2.1042234 \dots$$

The purpose of the authors to test the normality of these irrationals to different bases; their results and conclusions will appear elsewhere. For recent reviews on related matters, see [1], [2], [3] and the references cited there.

The three decimal numbers were compared with the slightly less accurate values in [2] in the vicinity of 22900D. No discrepancy was found. No details were given concerning programs or computer times, nor any explanation for the coincidence (?) that the number of digits to the base 6 turns out to be exactly 2^{15} .

D. S.

1. *Math. Comp.*, v. 21, 1967, pp. 258–259, UMT 17.
2. *Math. Comp.*, v. 22, 1968, p. 234, UMT 22.
3. *Math. Comp.*, v. 22, 1968, pp. 899–900, UMT 86.

46[7, 9].—DANIEL SHANKS & JOHN W. WRENCH, JR., *Calculation of e to 100,000 Decimals*, 1961. Computer printout deposited in the UMT file.

This calculation of e was performed seven years ago at the time that π was computed to the same accuracy [1]. In contrast to the latter computation, the programming for e had no special interest, inasmuch as it was based upon the obvious procedure of summing the reciprocals of successive factorials, and consequently it was dismissed in a footnote to [1].

Since a number of requests for copies of this approximation to e have been received, we accordingly deposit here two copies: the first, a full-size, 20-page, computer printout; the second, a photographic reduction thereof.

At the time of the computation, cumulative decimal-digit counts for $D = 10^3(10^3)10^5$ were tabulated, and nothing unexpected was observed. The final counts for $e - 2$ and $\pi - 3$ are as follows.

	0	1	2	3	4
e	9885	10264	9855	10035	10039
π	9999	10137	9908	10025	9971
	5	6	7	8	9
e	10034	10183	9875	9967	9863
π	10026	10029	10025	9978	9902

AUTHORS' SUMMARY

1. DANIEL SHANKS & JOHN W. WRENCH, JR., "Calculation of π to 100,000 decimals," *Math. Comp.*, v. 16, 1962, pp. 76-99.

47[7].—FREDERIC B. FULLER, *Tables for Continuously Iterating the Exponential and Logarithm*, ms. of 30 typewritten pages, 29 cm. Deposited in UMT file.

The theory of the continuous iteration of real functions of a real variable has been presented by a number of writers, including Bennett [1], Ward [2], and the present author [3].

The unique tables under review give 6D values of the continuously iterated function $F(x)$ and its inverse $G(x)$ for $x = 0(0.001)1$, with first differences, and for $x = 1(0.1)3$, without differences. Here $F(x)$ represents the exponential of zero iterated x times. Typical values for integral values of x are $F(0) = 0$, $F(1) = 1$, $F(2) = e$, and $F(3) = e^e$.

An introduction of five pages provides details of the procedures followed in the calculation of these tables. Appended notes explain how the tables can be extended in both directions with respect to the argument and include a discussion of the effect of the F operator on the number system of algebra.

It seems appropriate to mention here a similar study of Zavrotsky [4], which, however, led to radically different tables.

J. W. W.

1. A. A. BENNETT, "Note on an operation of the third grade," *Ann. of Math.*, v. 17, 1915-1916, pp. 74-75.

2. MORGAN WARD, "Note on the iteration of functions of one variable," *Bull. Amer. Math. Soc.*, v. 40, 1934, pp. 688-690.

3. MORGAN WARD & F. B. FULLER, "The continuous iteration of real functions," *Bull. Amer. Math. Soc.*, v. 42, 1936, pp. 393-396.

4. A. ZAVROTSKY, "Construccion de una escala continua de las operaciones aritmeticas," *Revista Ciencia e Ingenieria de la Facultad de Ingenieria de la Universidad de los Andes*, Mérida, Venezuela, December 1960, No. 7, pp. 38-53. (See *Math. Comp.*, v. 15, 1961, pp. 299-300, RMT 63.)

48[8].—JOHN R. WOLBERG, *Prediction Analysis*, D. Van Nostrand Co., Princeton, N. J., 1967, xi + 291 pp., 24 cm. Price \$10.75.